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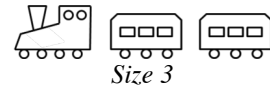
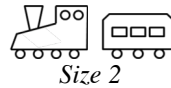
# Toy Trains

This problem gives you the chance to:

- find and use a number pattern
  - find an algebraic expression for a number pattern
- 

Brenda's toy shop sells toy trains.

A *size 1* set is just an engine, a *size 2* has an engine and 1 carriage, a *size 3* has an engine and 2 carriages and so on.



The engine has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14			

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.
2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

\_\_\_\_\_

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

\_\_\_\_\_

\_\_\_\_\_

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size  $n$  set.

\_\_\_\_\_

## 2009 Rubrics Grade 7

Toy Trains	Rubric													
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> <li>• finding and using a number pattern</li> <li>• finding an algebraic expression for a number pattern</li> </ul> <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points												
<p>1. Gives correct answers:</p> <table border="1" data-bbox="334 680 1269 747"> <tr> <td>Size of train set</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>Number of wheels</td> <td>8</td> <td>14</td> <td><b>20</b></td> <td><b>26</b></td> <td><b>32</b></td> </tr> </table> <p><i>Partial credit</i> One error</p>	Size of train set	1	2	3	4	5	Number of wheels	8	14	<b>20</b>	<b>26</b>	<b>32</b>	2  (1)	2
Size of train set	1	2	3	4	5									
Number of wheels	8	14	<b>20</b>	<b>26</b>	<b>32</b>									
<p>2. Gives correct answer: <b>74</b> Shows correct work such as: <math>8 + 11 \times 6</math> or continues table.</p>	1  1	2												
<p>3. Gives correct answer: <b>No</b> Gives correct explanation such as: <math>42 - 8 = 34</math> is the number of wheels for the carriages and this does not divide by 6. Accept: set 7 has 44 wheels and set 6 has 38 wheels.</p>	1  1	2												
<p>4. Gives correct answer such as: <b><math>6n + 2</math></b> or equivalent</p>	1	1												
<b>Total Points</b>		<b>7</b>												

## Toy Trains

Work the task and look at the rubric. What are the big mathematical ideas that a student needs to understand to be successful on this task? \_\_\_\_\_

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Look at student work for part 2, finding the number of wheels in train set #12. How many of your students:

- Solved the task correctly (74)? \_\_\_\_\_
- Gave a response of 80? \_\_\_\_\_ What misconception might be behind this solution?
- Gave a response of 76? \_\_\_\_\_ What misconception might be behind this solution?
- What strategies did students use?
  - Drawing and counting? \_\_\_\_\_
  - Continuing a table? \_\_\_\_\_
  - Using a number sentence (e.g.  $8 + 11 \times 6 =$ )? \_\_\_\_\_
  - Other? \_\_\_\_\_
- Which strategies are more useful for helping a student move to an algebraic expression?

Now look at student work for part 3, could Mick have 42 wheels. How many of your students could:

- Make a convincing argument for why 42 was incorrect? \_\_\_\_\_
- Did they back up their argument with other possibilities such as 38 or 44?
- Tried to explain that more were needed to make train #12? (still talking about part 2) \_\_\_\_\_
- Thought all trains past #7 would have at more than 42? (Weren't trying to get an exact amount of wheels) \_\_\_\_\_
- What other errors do you see in student thinking?

How did students approach part 3. Did they continue the table? \_\_\_\_\_ Write out a number sentence? \_\_\_\_\_ Use inverse operations ( - 8, then divide by 6)? \_\_\_\_\_ Other? \_\_\_\_\_

Finally, look at the algebraic expressions in part 4. How many of your students put:

$6n + 2$ or $6(n-1) + 8$	$6n + 8$	$n=6$	$6n + 86$	$n+6$	No response	Other

What are some of the misconceptions behind these errors?

What opportunities have students had to write algebraic expressions? What types of experiences help students to be able to describe patterns algebraically? What do they need to understand about operation?

## Looking at Student Work on Toy Trains

Student A uses a table to solve for part 2. But then the student plays around with numbers to see how to get the numbers in the table. This experimenting helps the student write a correct equation in part 5. The student makes a convincing argument for why 42 is not possible giving numbers near to 42 that are possible.

### Student A

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.
2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

6	7	8	9	10	11	12
38	44	50	56	62	68	74

$$\begin{array}{r} 12 \\ \times 6 \\ \hline 72 + 2 = 74 \end{array}$$

74 wheels

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

No.  $6+6+6+6+6+6+8 \neq 42$  He would have to have either 38 or 44 wheels

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

S = set #

w = # of wheels

$$(S \times 6) + 2 = w$$

$$(1 \times 6) + 2 = 8$$

$$(2 \times 6) + 2 = 14$$

7

Student B has a number rule to solve for the largest train size. The student then plays with the rule to see if the results match the numbers in the table. Notice how the student uses inverse operations to show why 42 does not work. Unlike many students, B is able to see that the  $6x$  used in part 2 is really the size number  $- 1$ . Noticing that there is 1 car that is not exactly 6 is very important when writing the algebraic expression.

**Student B**

The engine has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

$$\begin{array}{r}
 4 \quad 5 \quad 3 \\
 8+6x \quad 8+6x \quad 8+6x \\
 8+6 \cdot 3 \quad 8+6 \cdot 4 \quad 8+6 \cdot 2 \\
 8+18 \quad 8+24 \quad 8+12 \\
 26 \quad 32 \quad 20
 \end{array}$$

- Fill in the table to show how many wheels sets 3, 4 and 5 have.
- The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$$\begin{array}{r}
 8+6x \\
 8+6 \cdot 11 \\
 8+66 \\
 74 \text{ wheels}
 \end{array}$$

$\frac{12}{11}$  engine carriage

74 wheels

- Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

$$\begin{array}{r}
 8+6x=42 \\
 -8 \\
 \hline
 6x=34 \\
 x=5.66
 \end{array}$$

$$\begin{array}{r}
 32 \quad 38 \\
 30 \quad 36 \\
 \hline
 38 \quad 44
 \end{array}$$

no

I know because a size 6 would have 38 wheels and a size 7 would have 44 wheels.

- The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size  $n$  set.

Key  
size =  $s$

$8+6(s-1)$

7

7

Student C also notices that there are less cars with 6 than the total number of cars. The student understands the process and the pattern, but is not yet comfortable with algebraic notation.

**Student C**

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$12 - 1 = 11 \times 6 = 66$   
 $66 + 8 = 74 \text{ wheels}$  ✓ ✓

②  
 $\frac{74}{\quad}$  ✓

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

a size 5 train set has 32 and each train set increases its wheels by 6  
 so a size 6 train set has 38 and 7 has 44  $44 \neq 42$  ✓

②  
 $\frac{NO}{\quad}$  ✓

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$n = \text{size of train set} - 1 \times 6 + 8 \longrightarrow$

Student D uses repeated addition to solve for part 2. This takes away the necessity of finding the number of times the 6 is used and leads to the error in writing the algebraic expression in part 4. *What prompts or questions might you pose to push the thinking of this student to the next level?*

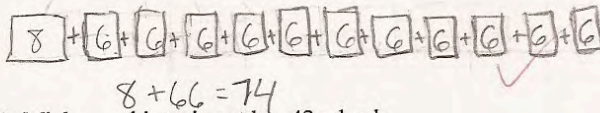
**Student D**

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.
2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.



3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

Because with a 5 train set there is 32 wheels so it doesn't make sense if a train set is 12 and with 42 wheels.

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$y = 6x + 8$

Student E knows that adding by 6 is like multiplying but fails to notice that the first car is counted twice in part 2. The student does not translate the rule into algebraic notation because the student doesn't understand the difference between a constant (6 every time) and another variable. *How do we help students understand this distinction? Do we ever push students with questions about how many variables are needed?*

**Student E**

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20 ✓	26 ✓	32 ✓

(2) 2

- Fill in the table to show how many wheels sets 3, 4 and 5 have.
- The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$8 + 6 \cdot 12 = 80$  ✓  
 $8 + 72 = 80$  ✓

$80 \times 0 = 0$  ✓

- Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

Because you cannot get 42 when you try to solve ~~8 + 6 \cdot 5~~ which equals 38 ~~or~~ or 8 + 6 \cdot 6 which equals 44. You cannot get 42 with the number of trains and

No ✓ 1

- The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$8 + x \cdot y$  ✓

7  
Toy Trains  
cannot get 42 with the number of trains and carriages provided (4)

(4)



Student F makes a drawing to help with the thinking in part 2. However by relying on the picture the student seems to only count 3 wheels per train for 12 cars, not the 11 drawn. *Drawing is time consuming. How can we help students move to more accurate and efficient strategies?* Notice that in part 4 the student multiplies by 4 instead of 6 or 3. Also the student tries to solve for a specific case, interpreting any number as pick a number. The student is not familiar with the idea of generalization.

**Student F**

The engine has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.

2. The biggest set in the shop is size 12.  
How many wheels does the size 12 set contain?  
Show how you figured it out.

3. Mick says his train set has 42 wheels.  
Can Mick be correct?  
Explain how you know.

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.  
Write an algebraic expression for the number of wheels in a size n set.

*Handwritten notes and calculations:*  
 - Vertical addition:  $\begin{array}{r} 6 \\ 12 \\ + 6 \\ \hline 18 \\ + 8 \\ \hline 26 \end{array}$ ,  $\begin{array}{r} 6 \\ 6 \\ + 6 \\ \hline 18 \\ + 8 \\ \hline 26 \end{array}$ ,  $\begin{array}{r} 6 \\ 6 \\ + 6 \\ \hline 18 \\ + 8 \\ \hline 26 \end{array}$   
 - Horizontal addition:  $12 \times 3 = 36$ ,  $36 + 8 = 44$   
 - Diagram: A train set with 12 cars, each with 3 wheels, and an engine with 8 wheels. Total wheels = 44.  
 - Algebraic expression:  $25 \times 4 + 8 = 108$

Student G is able to fill out the chart and find the wheels in a size 12 train set. The student does not show work to support the claim in part 3. *What might have improved this student's explanation? With so little work shown it is difficult to interpret  $n = y + x$ . What might the student be thinking? Do you think this student understands the multiplicative relationship in this pattern? What do you think the student knows about how the pattern works?*

**Student G**

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

$(2)^2$

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.
2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

9 74 ✓

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

NO ✓

ARE chart shows there is no 42.

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$N = y + x$

7

Student H does not understand the relationship of  $s$  to the number of carriages. *What prompts help students to notice the idea of a pattern not starting on the first term? How do we help students learn to quantify a relationship that doesn't include every item in the set?*

**Student H**

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	30 <del>x</del>	26 ✓	32 ✓

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.
2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$$8 + 6s = w$$

$$8 + (6 \cdot 12) = w$$

$$8 + 72 = 80 \quad \checkmark$$

$s$  = size number  
 $w$  = number of wheels

$$\frac{80 \times}{\quad} \times 0 = 0$$

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

$$8 + 6s = 42$$

$$\begin{array}{r} 8 + 6s = 42 \\ -8 \phantom{=} \\ \hline 6s = 34 \\ \frac{6s}{6} = \frac{34}{6} = 5\frac{2}{3} \end{array}$$

Mick is not correct  
because when you solve for the set number do not come out with a whole number so, it can not be a set size.

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size  $n$  set.

$$8 + 6s = w$$

$s$  = size number  
 $w$  = number of wheels

7

Student I is able to write a set of calculations that show understanding of the pattern in part 2. This set of calculations should be easily translatable into algebraic notation if the student had some familiarity with using symbols. *What questions could help the student think through writing the generalization?* In part 3 the student is still thinking about size 12 train set rather than is there any number that will be exactly 42 wheels. *How do we help students to think more globally as they work on tasks? Do students get enough opportunities to work with longer chains of reasoning? Where would you go next with this student?*

**Student I**

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5
Number of wheels	8	14	20	26	32

- Fill in the table to show how many wheels sets 3, 4 and 5 have.
- The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

3. 
$$\begin{array}{r} 11 \leftarrow \text{carriage} \\ 6 \leftarrow \text{wheels} \\ \hline 66 \\ 13 \leftarrow \text{engine} \\ \hline 79 \end{array}$$

$$\begin{array}{r} 79 \\ \hline \end{array}$$

- Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

No. Because eleven times 6 is 66, ~~x~~ ~~x~~ ~~0~~

- The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$$n = 7$$

Student J is able to use repeated addition or multiplication to extend the table. The student misinterprets the question in part 3. The student thinks that if there can be 74 wheels, then there must be 42 wheels. The student doesn't think is there some train set with exactly 42 wheels. In part 4 the student makes a number sentence using numbers in the given and the variable. But the expression does not relate to the calculations from earlier parts of the task.

### Student J

The table shows the number of wheels on each size of train set.

Size of train set	1	2	3	4	5	6	7	8	9	10	11	12
Number of wheels	8	14	20	26	32	38	44	50	56	62	68	74

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.

+ 2

(2)

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$$\begin{array}{r} 12 \\ \times 6 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 74 \\ \hline \end{array}$$

+ 2

(2)

3. Mick says his train set has 42 wheels.

Can Mick be correct?  
Explain how you know.

yes, because if 12 is 72, then anything below can have 42 wheels. yes X 0

4. The factory where the trains are made needs a rule for the number of wheels in any size set so that it can use this in its computer.

Write an algebraic expression for the number of wheels in a size n set.

$$806 - n$$

<b>Student Task</b>	Finding and using a number pattern. Finding an algebraic expression for a number pattern.
<b>Core Idea 3 Algebra and Functions</b>	<p><b>Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.</b></p> <ul style="list-style-type: none"> <li>• Represent, analyze, and generalize a variety of functions including linear relationships.</li> <li>• Relate and compare different forms of representation for a relationship including words, tables, graphs in coordinate plane, and symbols.</li> <li>• Express mathematical relationships using expressions and equations.</li> </ul>

*Mathematics of this task:*

- Recognizing and extending a pattern
- Using inverse operations to make a convincing argument about why 42 is not part of the answer set
- Writing an algebraic expression to describe the pattern for any solution in the set
- Noting that the first term has a different pattern than the rest of the terms

*Based on teacher observations, this is what seventh graders know and are able to do:*

- Identify pattern and complete the table
- Explain why 42 wheels are not possible
- Use repeated addition and extending the table to find the number of wheels in a size 12

*Areas of difficulty for seventh graders:*

- Writing an algebraic rule
- Recognizing that the 6 is multiplied by one less than the pattern number or double counting the engine
- Working with inverse operations
- Using repeated addition rather than seeing the multiplicative relationship
- Writing expressions with too many variables, not understanding the role of the “6”

*Strategies used by successful students:*

- Extending the table
- Using rules with multiplication
- Playing with number combinations and checking it with answers in their table

## MARS Test Task 1 Frequency Distribution and Bar Graph, Grade 7

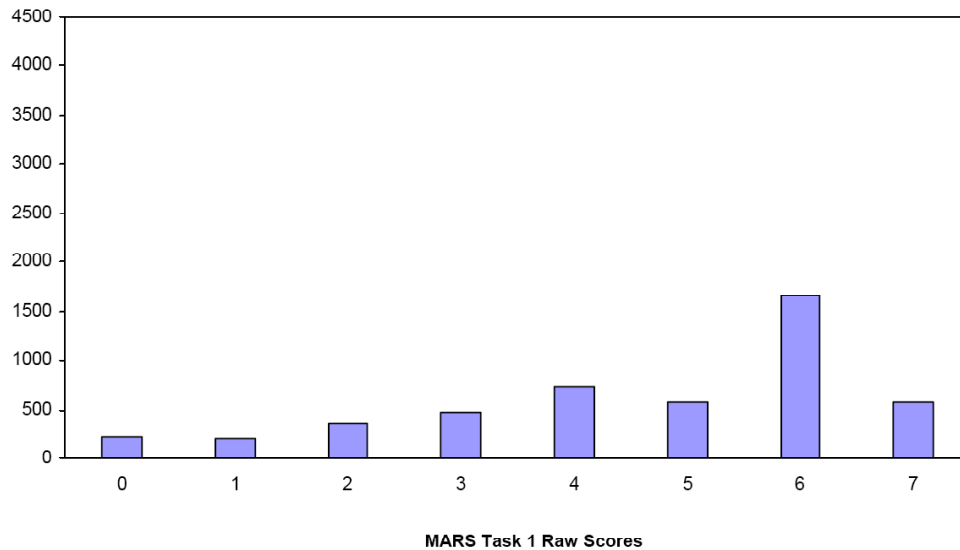
### Task 1 - Toy Trains

Mean: 4.63      StdDev: 1.92

Table 35: Frequency Distribution of MARS Test Task 1, Grade 7

Task 1 Scores	Student Count	% at or below	% at or above
0	213	4.5%	100.0%
1	198	8.6%	95.5%
2	351	15.9%	91.4%
3	472	25.8%	84.1%
4	726	41.0%	74.2%
5	572	53.0%	59.0%
6	1669	87.9%	47.0%
7	579	100.0%	12.1%

Figure 44: Bar Graph of MARS Test Task 1 Raw Scores, Grade 7



*The maximum score available for this task is 7 points.*

*The minimum score needed for a level 3 response, meeting standards, is 4 points.*

Most students, 91%, could recognize the pattern and extend the table. Many students, 74%, could also explain why 42 wheels were not possible. Almost half the students, 47% could recognize and extend the pattern using the table or the set number and explain why 42 was not possible. 12% could meet all the demands of the task including writing an algebraic expression to represent the pattern. Almost 5% of the students scored no points on this task. All the students in the sample with this score attempted the task.

## Toy Train

Points	Understandings	Misunderstandings
<b>0</b>	All the students in the sample with this score attempted the task.	Students made errors extending the pattern in the table.
<b>2</b>	Students could recognize and extend the pattern in the table.	Students had difficulty explaining why 42 was not possible. 7% thought that lots of trains had 42 or more wheels. They didn't think about exactly 42. Some students tried to give a verbal explanation of train set #12.
<b>4</b>	Students could use the table and explain why 42 was not a possible combination.	Students had difficulty with train set #12. 8% counted the engine as both 8 and 6 (80 wheels). Some students had answers of 74.
<b>6</b>	Students could recognize and extend the pattern using the table or the set number and explain why something wasn't possible.	Students struggled with putting together an algebraic expression. 20% wrote the expression $6n + 8$ . 8% wrote the expression $n + 6$ . 4% wrote $n=6$ . 12% did not attempt to write an expression.
<b>7</b>	Students could recognize and extend the pattern using the table or the set number and explain why something wasn't possible. Students could also describe the pattern by writing an algebraic expression. Students with this score did not use drawing and counting for parts 2 or 3. They saw the multiplicative nature of the pattern.	



## **Implications for Instruction**

Students at this grade level need frequent opportunities to work with patterns and try to describe them algebraically. When looking at patterns asking students questions, such as, “How does the pattern grow? What stays the same? What changes? Is there anything different in the first case?” helps students focus on the attributes of the pattern. Too often students rush to getting an answer by drawing and counting or using repeated addition. These strategies don’t serve to notice the multiplicative thinking or ideas about calculation patterns that can then be translated into algebraic terms. They are counter-productive to the new thinking we want students to develop at this grade level. Having students create good verbal descriptions of what is happening and of their description of the calculation process to extend the pattern helps them to create generalizable rules.

## **Ideas for Action Research**

Often the focus is on the misconceptions of students. But for this exercise, you and your colleagues should focus on the work of successful students. What strategies did they use? How is their thinking process, even in the early parts of the problem, different than less successful students?

Gather together the entire perfect papers, 7 points. Look through the work and see what commonalities appear in their strategies for part 2 extending the pattern beyond the table to a size 12 train set. How does this work or way of tackling the problem show the beginnings of algebraic habits of mind? How do their solution strategies in part 2 aid in making the generalization in part 5? What are possible questions they are asking themselves to help them with the calculations in part 2? What attributes of the pattern help them with the calculations? How can you use those questions to help other students develop a similar “self-talk”?

When you are done, look at the research from the sample on the next page.

In the sample there were 4 strategies for solving question 2.

- $12-1 = 11, 11 \times 6 = 66, 66 + 8 = 74$  *Used by 20%*
- $12 \times 6 = 72, 72 + 2 = 74$  *Used by 10%*
- $8 + 11 \times 6$  *Used by 70%*
- $8 + 6+6+6+6+6+6+6+6+6+6+6+6+6=74$  *Used by 10%*

Can you give a reason for each step?

How did you know what to multiply by 6 or how many 6's to add?

Why did you add 2? Where did the 2 come from?

Why did you add 8? Where did the 8 come from?

One student wrote the algebraic expression  $8 + 6n - 6$ . Why does this make sense?

These questions can help push students to really clarifying their thinking. These bare arithmetic problems are good to share with the whole class to see if they can make sense of the process.

Below are some examples from students with 7's.

**Student 1**

The table shows the number of wheels on each size of train set.

X	Size of train set	1	2	3	4	5
Y	Number of wheels	8	14	20	26	32

$+6$   $+6$   $+6$  (2)

1. Fill in the table to show how many wheels sets 3, 4 and 5 have.

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

Handwritten work for question 2:

$$\begin{array}{r} 8 \\ + 6 \\ \hline 14 \\ + 6 \\ \hline 20 \\ + 6 \\ \hline 26 \\ + 6 \\ \hline 32 \end{array}$$

12 24 48  
12 24 74  
12 20 6

74 wheels ✓

(2)

3. Mick says his train set has 42 wheels.

**Student 2**

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

Handwritten work for question 2:

$12 - 1 = 11$   
↑ set# ↑ engine

carriages →  $11 \cdot 6 = 66 + 8 = 74$   
↑ wheels per carriage

Rule:  $y = (x-1) \cdot 6 + 8$

74 ✓ (2)

### Student 3

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

$$\begin{array}{r} \times 11 \\ 6 \\ \hline 66 \end{array}$$

Engine = 8 wheels  
11 Carriages = 66 wheels  
74

74 wheels

### Student 4

2. The biggest set in the shop is size 12.

How many wheels does the size 12 set contain?  
Show how you figured it out.

6	7	8	9	10	11	12
38	44	50	56	62	68	74

$$\begin{array}{r} 12 \\ \times 6 \\ \hline 72 + 2 = 74 \end{array}$$

74 wheels

3. Mick says his train set has 42 wheels.

Performance Assessment Task
Toy Trains Grade 7
This task challenges a student to use algebra to represent, analyze, and generalize a variety of functions including linear relationships. A student must be able to relate and compare different forms of representation for a relationship including words, tables, graphs, and writing an equation to describe a functional pattern. A student must be able to use rules of operations to extend a pattern and use its inverse.
Common Core State Standards Math - Content Standards
<p><b><u>Expressions and Equations</u></b></p> <p><b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b></p> <p>7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate, and assess the reasonableness of answers, using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50 for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i></p> <p>7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>a. Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x+q) = r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of operations used in each approach. <i>For example the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i></p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.4 Model with mathematics.</b></p> <p>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p> <p><b>MP.8 Look for and express regularity in repeated reasoning.</b></p> <p>Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation <math>(y - 2)/(x - 1) = 3</math>. Noticing the regularity in the way terms cancel when expanding <math>(x - 1)(x + 1)</math>, <math>(x - 1)(x^2 + x + 1)</math>, and <math>(x - 1)(x^3 + x^2 + x + 1)</math> might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process,</p>

while attending to the details. They continually evaluate the reasonableness of their intermediate results.

### Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2009	7	4	74%